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Developing Ethnomathematical-Based Learning Stages on Mathematical Communication Skills with Kastolan Error Analysis

Abstract: Ethnomathematics-based learning has been effective for students' mathematical communication skills but can still be optimized again by minimizing mistakes that students often make. So the objectives of this study are: 1) to analyze student errors in solving mathematical communication problems in terms of student ability categories; 2) analyze the Ethnomathematics-based learning process with the help of the Geogebra application; 3) develop Ethnomathematics-based learning steps based on student error analysis. The study subjects were randomly selected, namely 36 students of junior high school class IX odd semester academic year 2023/2024 SMP Negeri 1 Tanara. Data were obtained from mathematical communication skills tests, interest questionnaires, observation sheets, and interview sheets. Data analysis techniques start with data analysis, tests of students' mathematical communication skills with qualitative data analysis, then to interpret research findings, followed by quantitative data analysis. The analysis of students' errors in solving mathematical communication problems in terms of student ability categories found that students in medium and low ability categories made more procedural errors. At the same time, students in the high-ability category tend not to make significant mistakes. The results of the triangulation found two stages of learning needed to minimize student errors, namely the exercise stage and the investigation stage. The stages of Ethnothematics-based learning consist of exploration, mapping, explanation, exercise, investigation, and reflection. The effectiveness of the learning model developed on students' mathematical communication skills will be analyzed for further research.

Keywords: castellan analysis, ethnomathematics, geogebra, exercise stage and research stage

INTRODUCTION

Early mathematicians did not explicitly state that culture had nothing to do with mathematics. However, in the last two decades, D'Ambrosio and Milton Rosa of Brazil popularized the cultural link to mathematics. Initially, mathematics taught in schools was free from culture, meaning that mathematics was free from the cultural emphasis that occurred in the social dynamics of traditional or modern society (D'Ambrosio & Rosa, 2017; Rosa & Orey, 2011). Many people think that mathematics is kept away from culture, but this assumption is not all true because mathematics fundamentally contributes significantly to developing a culture that occurs in society (Freudenthal, 2006). For example, in Banten culture, various calculations are still carried out, which turns out to be closely related to mathematics itself. For instance, in Patingtung art culture, to measure sound, the terms "pa", "ting", and "tung" refer to logic and mathematical patterns in measuring Titi scale and rhythm, so from the statement that we should argue that between culture and mathematics, there is a relationship (Albanese & Perales Palacios, 2015; Rosa & Orey, 2011)

Ethnomathematics is the study of mathematics (mathematical ideas) and its relationship with culture in the context of social life in society (Gerdes, 2010). This is in line with Albanese & Perales Palacios's (2015) and Rosa & Orey (2011) opinion, that ethnomathematics is a study that examines mathematical ideas or practices in various cultures that show the interrelationship between the two. Ethnomathematics is an approach to teaching and learning mathematics that builds on students' prior knowledge, background, the role the environment plays in terms of content and methods, and past experiences and their current environment (D'Ambrosio, 1990). Therefore, the development of culture in mathematics is increasingly

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becoming a hotly discussed issue and several studies have applied ethnomathematics in the concept of learning mathematics in the classroom. Ethnomathematical developments developed among them were introduced by Milton Rosa and Orey, namely ethnomodeling. Ethnomodeling is the study of a culture's ideas and procedures viewed from the mathematical modelling lens (Rosa & Orey, 2013). This is confirmed by Orey & Rosa (2011), who state that mathematical modelling involves practices that occur in the culture. Therefore, it is necessary to understand well that every culture that occurs in an area is not only extracting mathematical ideas related to that culture but also trying to connect about ethnomodeling, which can be seen from the perspective of academics.

The development of ethnomathematics develops in the process of learning mathematics. This is because ethnomathematical learning that develops in culture in an area will strengthen students with their customs and environment (D'Ambrosio, 2007; Knijnik, 2002; Mosimege, 2012; Prieto et al., 2015). This will make learning richer, and students can interpret the culture around them in the context of mathematics so that students' love for their area arises (D'Ambrosio, 1985; D'Ambrosio, 1999; D'Ambrosio & Rosa, 2017). Mathematics and culture will be an interesting scientific context because students will learn mathematics based on the cultures they understand that are relevant to their daily lives. Thus, students can more easily understand mathematics through acculturation between mathematics and culture. Banten is one of the regions that has cultural diversity that is still developing today, for example in traditional arts, traditional houses, typical foods, traditional clothing, and so on. The cultural potential in Banten must be explored optimally because Banten culture has its uniqueness and distinctiveness. This uniqueness can be an asset for the existence of Banten culture to be introduced to the general public.

In the past, there was a sultanate called the Banten Sultanate in Banten. After the Banten sultanate ended, what remains today are historical relics in the form of former royal palaces, the Great Mosque of Banten, Fort Speelwijk, Kasunyatan Mosque, and so on. These historical buildings belong to a cultural category of artefacts that have a basis in mathematical concepts (Anggraheni et al., 2020). In the Banten region, a community, namely the Baduy Tribe, still upholds the traditions inherited from its ancestors. According to Sutoto (2017), Baduy or kanekes are indigenous Sundanese people in Lebak Regency, Banten. The Baduy tribe has several types of culture, including the Baduy traditional house (Susilowati et al., 2020), motif batik Baduy, tenun Baduy (Namirah et al., 2019), system leuit Baduy (Iskandar & Iskandar, 2017), and totopong Baduy (Isnendes, 2014). Objects such as the roof of a traditional Baduy house, typical Banten batik motifs, typical Banten gates, Kasunyatan mosque gates, Baduy totopong, Banten tower pedestals, Baduy weaving ulurs, the roofs of the great mosque of Banten, and leuit Baduy contain various mathematical concepts. Many mathematical concepts can be studied in Banten culture, so much research is done to improve students' understanding of the material presented.

Various studies on Ethnomathematics in Banten culture have been carried out, including Karinawati et al. (2016), namely learning Sundanese ethnomathematics (Banten culture) look very interested so that students are active when working on the LKS given. One of the activities carried out in learning is using congklak and wayang Cepot games, which are very familiar to students, so that students are brave and confident to convey the results of their discussions or express their ideas. The research results (2021) show that one of the motifs of Banten batik is the philosophy of kahirupan baduy, which is the daily activity of the Baduy tribe community. There is a concept to find the area and circumference of the motif from the Kahuripan baduy motif in the form of an isosceles triangle. Furthermore, the lebak batik motif also contains geometric aspects and mathematical ideas in batik making activities such as symmetry at points, symmetry at lines, rectangles, triangles, and star graphs. Trisnawati (2022) also researched the ethnomathematics of Banten culture and it is known that an

ethnomathematical approach based on local culture in Banten can be used as an alternative choice on the subject of rows and rows. The results of his research also show that students can build mathematical concepts and see abstract concepts becomes more accessible because it starts from concrete things they know. Furthermore the results of Nirmalasari et al. (2021) research, show that Banten culture has various Pythagorean concepts, besides that, according to him, with ethnomathematical studies of Banten culture as a form of strengthening character education on nationalist values by having competence, love for their own culture can grow and be owned by students as the identity of the nation.

In research that has been done, ethnomathematics is used to convey teaching material to be more contextual and something they are familiar with. Rosa & Orey (2021) mentioned that each region has a different culture, so each region has different ways of communicating, reading, and interpreting the diverse world. Furthermore, Ethnomathematics is used to translate cultural mathematical ideas into academic mathematics (Umbara et al., 2021). So that the learning design developed with Ethnomathematics will be maximized to streamline students' mathematical communication skills. On the other hand, learning mathematics requires mathematical communication skills. National Council of Teachers of Mathematics (2000) and Baiduri et al. (2020) wrote, that mathematical communication skills are a way for students to express mathematical ideas orally, in writing, pictures, diagrams, using objects, presenting in algebraic form, or using mathematical symbols. According to NCTM, one of the basic mathematical abilities students must have in mathematics is mathematical communication skills.

Furthermore, Baroody & Coslick (1993) also mentioned that there are at least two reasons why mathematical communication is essential, namely: (1) mathematics as language, meaning that mathematics is not just a thinking aid but mathematics helps to find patterns, solve problems, and communicate various ideas precisely and concisely; (2) mathematics is learning as social activity, meaning mathematics is a social activity in mathematics learning, as well as interaction between students and teacher communication with students. Communication between teachers and students is essential to learning mathematics to guide students to understand concepts or find solutions to problems. Through communication, students can exchange and explain their ideas or understandings to their friends (Hendriana et al., 2013). A similar statement was expressed by Clark et al. (2005) who explained how important it is to have mathematical communication skills because mathematical communication is a way and understanding to construct meaning and explain one's ideas. Mathematical communication is one of the standard processes in learning Mathematics (CUMHUR & TEZER, 2020; Sumaji et al., 2019; Viseu & Oliveira, 2012).

Mathematical communication is one of the competencies students need in real life and learning Mathematics. In the classroom, students always communicate with teachers and other students to solve problems in mathematics problems and present mathematical solutions or ideas to others. However, there are many difficulties when they exchange information or communicate with others using the language of mathematics in class. Therefore, the ability to communicate Mathematics is one of the competencies students need. However, judging from the results of The Trends in International Mathematics and Science Study (TIMSS) 2015 ranked Indonesian students 45th out of 50 countries in mathematical communication, and Program for International Student Assessment (PISA) 2018 which ranked Indonesian students 74th out of 79 countries in mathematical communication.

Furthermore, based on the results of previous research, it is known that students' mathematical communication skills are still low. Österholm (2006) pointed out that students' mathematical communication skills were still considered low, especially the skill and accuracy of drama to observe or recognize a mathematical problem. Österholm's (2006) research results stated that respondents seemed to have difficulty articulating reasons in understanding a

reading. Low mathematical communication skills were also shown in the study Zulkarnain (2013), where students could not communicate ideas well. This is known from the results of students' answers that there are still wrong answers to the questions given and the calculation steps carried out by students have not been well organized and inconsistent. Students cannot yet fully provide arguments based on mathematical principles and concepts. Phuong & Tuyet (2018) wrote in her research that students are more interested in solving multiple-choice questions than questions that ask them to give opinions, so students only give short answers such as yes or no. According to him, this is because students feel afraid of being wrong and lazy in writing down their ideas, less accustomed to expressing their ideas in writing, students only do the same exercises repeatedly as in the exercise book so that when students are faced with different problems students do not know how to solve them.

When a student cannot describe their idea logically, this is because the student cannot communicate his thoughts in words so problems arise because the student cannot describe his mathematical idea coherently (Baxter et al., 2005). These problems certainly require attention so that solutions can be found to improve students' communication skills in mathematics. Improving students' mathematical communication skills must go hand in hand with the learning process. We can optimize communication skills by applying learning models that provide opportunities for students to discuss and interact with each other so that their mathematical communication skills improve (Tinungki, 2015). Pugalee (2001) further, that for students to be trained in mathematical communication skills, in learning students need to be accustomed to giving arguments for each answer and responding to answers given by others, so that what is being learned becomes more meaningful to him.

In this study, an analysis of student errors in solving mathematical communication problems was carried out after being given Ethnomathematics-based learning with the help of the geogebra application. By analyzing the tendency of mistakes made by students in solving mathematical communication problems after being given Ethnomathematical learning assisted by the Geogebra application, it is then used as a reference to develop stages in an effective Ethnomathematics-based learning model so that these errors do not repeat themselves and can optimally improve students' mathematical communication skills. Kastolan stage is one of the tools to analyze the description answer errors students make in solving questions. Kastolan's stages consist of conceptual, procedural, and technical errors (D et al., 2021; Najwa, 2021; Yarman et al., 2020).

Mistakes made by students can be information and reference regarding progress and what shortcomings still have to be learned and done during the Ethnomathematical learning process. Furthermore, learning steps are designed based on these references to minimize student errors and improve students' mathematical communication skills. So that the objectives in this study are: 1) analyzing student errors in solving mathematical communication problems after being given Ethnomathematics-based learning with the help of the Geogebra application; 2) analyze Ethnomathematics-based learning steps with the help of the Geogebra application on students' mathematical communication skills. The benefit of research is to develop Ethnomathematics-based learning with an effective learning design to be optimal for students' mathematical communication skills.

METHOD

This research uses quantitative and qualitative research methods, in the first stage using quantitative methods to obtain quantitative data then in the second stage using qualitative methods to deepen, expand, prove quantitative data. Quantitative research relies on the collection and analysis of numerical data to measure, describe, explain, or predict, as well as make broad generalizations (Mertler, 2020). While qualitative research is interactive research in which researchers engage in continuous and continuous experience with participants, this

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involvement will later raise a series of strategic, ethical, and personal issues in the qualitative research process (Creswell, 2015).

The study subjects were randomly selected, namely 36 students of junior high school class IX odd semester academic year 2023/2024 SMP Negeri 1 Tanara. The instruments used in this study were mathematical communication skills tests, interest questionnaires, observation sheets, and interview sheets. The mathematical communication skills test is in the form of 3 description questions that were previously tested first and have been declared valid by experts to be used. Mathematical communication skills tests are made based on indicators of mathematical communication skills, namely: 1) the ability to express mathematical ideas through, writing and demonstrating and describing visually; 2) the ability to understand, interpret and evaluate mathematical ideas either orally, in writing or other visual forms; 3) the ability to use terms, mathematical notations and structures to present ideas and describe relationships and models of situations (Niss & Højgaard, 2019). Student interest questionnaire in the form of 15 statements that have been validated by experts, made based on 5 indicators of learning interest, namely: 1) there is a feeling of pleasure towards learning; 2) there is a concentration of attention and thought on learning; 3) the willingness to learn; 4) there is a willingness from within to be active in learning; 5) there is an effort made to realize the desire to learn (Wulandari, 2021).

The preparation of the questionnaire instrument uses the Likert scale with 5 answer choices, namely always, often, sometimes, rarely, and never. Student interview sheets are not arranged systematically but in the form of outlines of problems that want to be explored, namely how students solve problems, mistakes students make in understanding and solving problems, and the learning process experienced during learning. The research flow is that learning is carried out in 8 meetings for 4 weeks in transformation geometry lessons. The essential competencies of transformation geometry chosen are explaining geometric transformations (reflection, translation, rotation, and dilation) related to contextual problems, and solving contextual problems related to geometric transformations. After learning, students are given a mathematical communication skills test and fill out a learning interest questionnaire. The data of the mathematical communication skills test results are processed and classified based on high, medium, low, and meager abilities.

Table 1. Result Category Percentage of Students' Mathematical Communication Skills

Category	Percentage (%)
High	$75 \leq P \leq 100$
Medium	$50 \leq P < 75$
Low	$25 \leq P < 50$
Very Low	$0 \leq P < 25$

The score percentage formula used is::

$$\text{Score percentage} = \frac{\text{number of scores obtained}}{\text{maximum number of scores}} \times 100\%$$

The data on the students' mathematical communication skills test is also described based on the type of error students solve each question item according to Kastolan, namely conceptual, procedural, and technical errors. Furthermore, students with high, medium, and low communication skills criteria were each selected two students as subjects to be interviewed about the difficulty of solving the problem and their responses regarding the learning process given.

Data analysis techniques start from data analysis, tests of students' mathematical communication skills with qualitative data analysis, then to interpret research findings,

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followed by quantitative data analysis. Error assessment on each indicator of mathematical communication skills is reviewed based on error indicators according to Kastolan adapted from (D et al., 2021; Najwa, 2021; Yarman et al., 2020).

Table 2. Error Analysis Indicator According to Kastolan

Error Type	Indicator
Conceptual Errors	1. Errors in the use of formulas in answering questions 2. Use a formula that does not match the conditions or prerequisites for the formula to apply
Procedural Errors	1. Inappropriate in performing troubleshooting steps 2. Not coherent in performing troubleshooting steps 3. Unable to solve the problem in its simplest form 4. Cannot manipulate troubleshooting steps
Technical Errors	1. Error in calculating the value of the calculate operation 2. Errors in writing, i.e. some constants or variables are miswritten or missed or errors moving constants or variables from one step to the next 3. Error in changing values to variables

The research model used at the stage of concluding is the Miles and Huberman Model, which consists of (Miles & Huberman, 1994):
1. Data reduction
    Research that has obtained data is written in reports or detailed data in data reduction. Reports prepared based on the data obtained are reduced, summarized, selected the main things, focused on the essential things. Data results that summarize and sort by units of concepts, themes, and specific categories will provide a sharper picture of the results of observations and make it easier for researchers to find data in addition to previous data obtained if needed.
2. Categorization
    Data presentation techniques in qualitative research can be carried out in various forms such as tables, graphs, and the like. The presentation of data can also be done in the form of short descriptions, charts, relationships between categories, flowcharts and the like. Thus, narrative text is most often used to present data in qualitative research. The function of data display in this study is to facilitate and understand what happens in analysing students' mathematical communication skills regarding student abilities.
3. Synthesis
    Synthesizing is looking for links between one category and another. The association of one category with another category is named/labeled again.
4. Drawing up a working hypothesis
    Conclusion drawing in this study is a presentation of the conclusions of the research results. The conclusions in qualitative research are initially provisional, and will change if there is no solid supporting evidence at a later stage of data collection. However, if the conclusions are supported by valid and consistent evidence when researchers return to the field when collecting data, then the conclusions put forward are credible. Conclusions in qualitative research can be new findings that have never existed before. Findings can be in the form of descriptions or descriptions of an object that was previously still unclear or dark so that the suit under study becomes clear, in the form of causal or interactive relationships, hypotheses or theories.
    In this study, researchers used a technique to check the validity of credibility data with a triangulation model. Triangulation is essentially a multimethod approach researchers take when collecting and analyzing data. The basic idea is that the phenomenon under study can be

well understood so that a high level of truth is obtained if approached from different points of view. Photographing a single phenomenon from different angles will allow a reliable level of truth to be obtained. Therefore, triangulation is an effort to check the correctness of data or information obtained by researchers from different points of view that differ by reducing as much as possible the differences that occur at the time of data collection and analysis. Triangulation includes four types, namely:

a. Triangulation method is done by comparing information or data in different ways.
b. Inter-researcher triangulation, which is triangulation is done by using more than one person in data collection and analysis.
c. Triangulation of data sources, which is to explore the truth of certain information through various methods and sources of data acquisition
d. Theory triangulation is the final result of qualitative research in the form of an information formulation or thesis statement.

However, here researchers use the type of source triangulation because it is more suitable to find answers from the purpose of the research conducted. Source triangulation is done by exploring the truth of certain information through various data acquisition sources. In this study, the data sources used were tests of students' mathematical communication skills (high, medium, and low), interview results, interest questionnaire results, and observations during the learning process (Kelle et al., 2019).

RESULTS AND DISCUSSION

The following data on students' mathematical communication ability test results are presented in tables based on high, medium, low, and shallow criteria after being given Ethnomathematics-based learning with the help of the Geogebra application.

Table 3. Student Mathematical Communication Skills Test Results

Categories Mathematical Communication Skills	n	Percentage (%)	Mean
High	7	19.44	85.71
Medium	20	55.56	70.33
Low	9	25	42.22
Total	36	100	66.30

The table above shows that of the 36 students who were treated with Ethnomathematics-based learning with the help of the geogebra application, more than 50% had moderate categories of mathematical communication skills. The rest is only 25% of students whose mathematical communication skills are in the low category and 19.44% of students whose mathematical communication skills are in the high category. The overall average is known to have exceeded the standard average of 65 which is 66.30. This data shows that ethnomathematics-based learning with the help of geogebra applications positively impacts students' mathematical communication skills. However, it is known that on average 55.56% of students in the medium category can still be improved to be able to reach the high category, as well as an average of 25% of students in the low category can still be improved again to be able to reach the medium or high category. In other words, the Ethnomathematics-based learning process with the help of the geogebra application can still be developed to be more optimal for students' mathematical communication skills. For this reason, further analysis of the stages during the learning process is carried out and analysis based on the types of mistakes students make, then triangulation is carried out based on the results of interviews, questionnaire results, and direct observation of the learning process that takes place. This is done to find the most optimal learning strategy for students' mathematical communication skills.

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Types of Student Errors in Solving Mathematical Communication Problems

The communication skills test questions consist of three description questions based on three mathematical communication skill indicators. The following are the assessment results on each indicator of mathematical communication skills reviewed based on error indicators according to Kastolan.

Table 4. The results of the Mathematical Communication Skills Test are reviewed from the Indicators and Types of Errors According to Kastolan

Indicator	Mean	Number of Students who make a tendency to make mistakes (n = 36)		
		Conceptual Errors	Procedural Errors	Technical errors
Indicator 1 the ability to express mathematical ideas through, writing and demonstrating and describing visually	4.42	8 (22.22%)	15 (41.67%)	12 (33.33%)
Indicator 2 the ability to understand, interpret and evaluate mathematical ideas both in writing and in other visual forms	2.11	15 (41.67%)	22 (61.11%)	19 (52.78%)
Indicator 3 ability to use mathematical terms, notations and structures to present ideas and describe relationships and models of situations	3.42	11 (30.56%)	17 (47.22%)	15 (41.67%)
Total		34	54	46

The data in table 4 shows that most students made procedural errors in solving mathematical communication problems of geometry transformation material. It is also known that most students make mistakes on question number 2 (indicator 2) with the lowest average score of mathematical communication skills at 2.11 out of a maximum score of 5. Question number 2 is a question that is classified as a problematic category when tested. The indicator in question 2 is the ability to understand, interpret and evaluate mathematical ideas in writing and other visual forms. The ability to interpret mathematical ideas is an ability that requires a deep understanding of concepts in order to be able to come up with ideas when facing mathematical problems (Rohid & Rusmawati, 2019). Further analysis is carried out based on student answer sheets and interviews with concerned students to obtain more in-depth information.

Student Answer Sheet Analysis

The following analysis is carried out on the answer sheet that students write in solving mathematical communication problems. The questions consist of 3 representing 3 indicators of students' communication skills. Each student's answer sheet was analyzed based on error indicators, namely conceptual errors, procedural errors, and technical errors and reviewed based on the student's mathematical communication skills category.

Table 5. Number of Errors Students Complete

Error Indicators	Categories Mathematical Communication Skills			Sum
	High	Medium	Low	
Konseptual	4	9	21	34
Prosedural	11	21	22	54
Technical	17	13	18	46
Sum	32	43	61	

Table 5 shows that students in the low category did the most on total errors while students in the high category did the least. However, in the technical error indicator column it can be seen that students in the category of mathematical communication skills make almost the same number of errors as students in the low category. Furthermore, students in the medium category made fewer errors in solving mathematical communication problems than students in the high category. From this data, it can be concluded that students with high communication skills are also very likely to make more mistakes than students with low communication skills. Thus, further analysis was carried out on student answer sheets selected based on the category of students' mathematical communication skills.

The following is question number 1 which represents indicator 1, namely the ability to express mathematical ideas through writing and demonstrate and describe visually.

Jember fashion Carnaval (JFC) is a work of art born and developed in Jember City. JFC was first held on January 1, 2003 around Jember city square. The 12th JFC in 2013 carried the theme of defile "Artechsion" (Art meet Technology and Illusion). By combining technology and art, culture produces extraordinary works. Here is an example of a JFC defile Betawi costume image.

Betawi Parade

The question: The sketch on the left wing of the JFC costume looks like the following image. Draw the following image mirroring results!
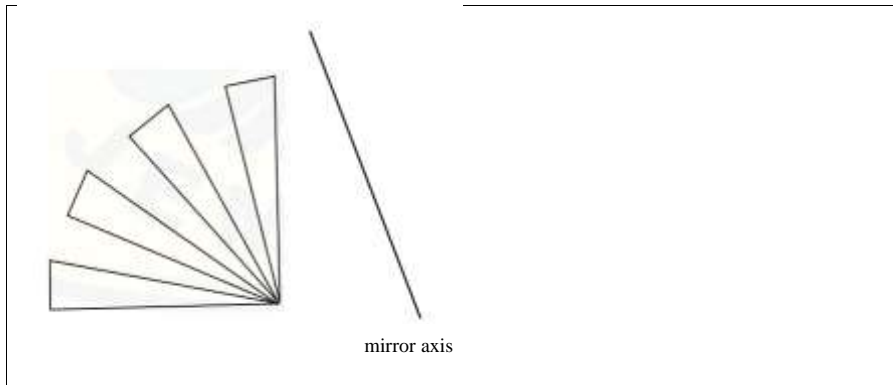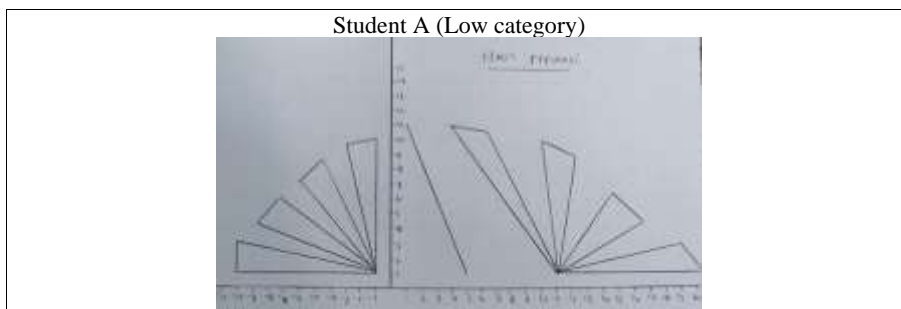
mirror axis

Figure 1. Question Number 1 Indicator 1 Mathematical Communication Skills

Furthermore, the results of students' work in solving question 1 are presented based on the categories of student communication skills (high, medium, and low) randomly selected in the following picture.

Student A (Low category)
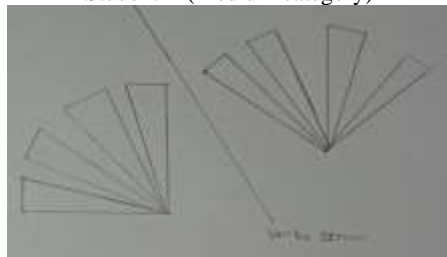

Error Description:
The picture on the side is an answer sheet of students who belong to the low category of mathematical communication skills. The mistake made by student A is a procedural error, which can be seen from the error in carrying out the problem-solving steps. The question given asks students to determine the results of mirroring based on the object and axis of the mirror given. Student A understands that mirroring an object will result in the same object. However, student A does not understand the rules in reflecting an object on the mirror axis. Student A's work results show that students are still shackled to the concept of mirroring objects on the mirror axis, which must be straight vertically parallel to the Y axis. So that the results of the reflection depicted seem to be forced parallel to the X axis but try to adjust the tilted mirror axis. As a result, the mirroring results made are incorrect. This fact can also be seen from the cartesian coordinate axis described by student A as the axis of help to solve the given problem. Even though the axis is not in the problem and does not have any impact on helping to solve the problem.

Results of the interview and its analysis:
Student A stated that he had no difficulty when solving question number 1. According to student A, the result of his work is already the correct answer. However, when asked, "if you look in the mirror is the exact mirror the same or maybe slightly different?" student A

replied confidently, "definitely it should be the same mom". This explains that student A understands the mirroring concept but still has difficulty in the mirroring procedure. Referring to indicator 1, namely the ability to express mathematical ideas through writing and demonstrate and describe visually, student A has a proven ability that is still relatively low in describing his ideas to solve given problems. This fact shows that students need improvement in their skills in carrying out problem solving steps in order and correctly. Furthermore, student A should get used to double-checking his worksheet when solving problems.

Student B (Medium category)

Error Description:
Student B gives an answer that looks correct regarding the direction of the results of mirroring the object against the correct mirror axis. However, analyzed in more detail, it is known that there are still many gaps that show that the reflection results are still incorrect. Student B showed a technical error seen from the image of the reflecting object which was smaller than the original. The concept of mirroring and the steps taken to reflect the object against the inclined axis are correct. If you pay attention, some shapes of the reflected objects appear thinner than the actual object. Even though the results of mirroring an object should be precisely the same as the object being mirrored.

Results of the interview and its analysis:
The interview revealed that student B seemed hesitant about the answer, and had stated that the answer was still incorrect. Student B stated that it was more comfortable to solve the problem in a group way, because there was a lot of input and discussion towards improving the answers to the questions being done. Furthermore, student B stated, "I know that the mirrored image I made is not accurate and could still be better, but the location is correct, ma'am". This statement indicates that student B understands the concept of mirroring, but is less careful and neat in describing the problem.

Student C (High category)
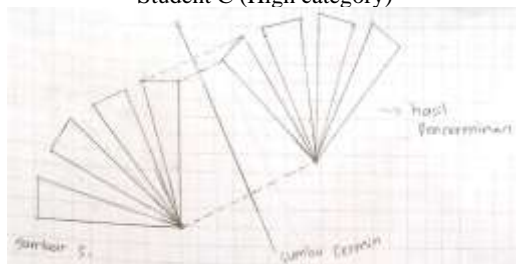
Error Description:
Student C answers correctly and adds auxiliary lines when reflecting the problem object. Student C shows that he understands the concept and understands the procedure, as well as using the proper technique.

Results of the interview and its analysis:
The interview results found that student C was already very confident in his answer. Interestingly, student C mentioned that the mirroring image he made applies to flat mirrors but not to convex mirrors. In fact, the teacher has never mentioned flat mirrors and convex mirrors during the learning process. That is, student C has more knowledge than others. When asked, "do you know where the convex mirror comes from"?. Student C replied "you know because at home I have a convex mirror". Student C stated that the mirroring material was entertaining, while doing homework student C shared that he tried to solve it using the mirror he had. However, the results of mirroring the object shown enlarged, apparently because the mirror used is convex. Student C's attitude shows high curiosity and willingness to explore. Willingness to explore and high curiosity are abilities that need to be instilled in other students to have better mathematical communication skills. Curiosity is significant in learning, and doing research in completing tasks makes learning more meaningful (Wagstaff et al., 2021).

Figure 2. Student Answer Description Question Number 1 is reviewed from the Student Communication Skills Category

Furthermore, question number 2 represents the second indicator, namely the ability to understand, interpret and evaluate mathematical ideas in writing and other visual forms. The problem is:

The following is an example image of one of the Tibet defile JFC costumes.

Question:
Consider the following sketch sketch of the wings of the Tibet defile costume.
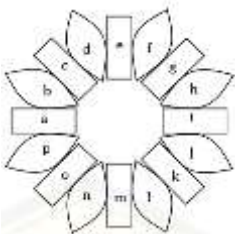
If the c shape is rotated 270º counterclockwise concerning the center of rotation, which shape results from the rotation?
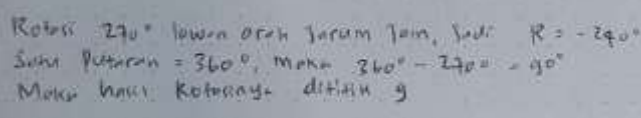
Picture 3. Question Number 2 Indicator 2 Mathematical Communication Skills

Furthermore, the results of students' work in solving question 2 are presented based on the categories of student communication skills (high, medium, and low) randomly selected in the following picture.

Student A (Low category)

Rotation 270º counterclockwise, so R = -240º
One box = 360º, mark 360º – 270º = 90º
Then the result of the rotation is at point g
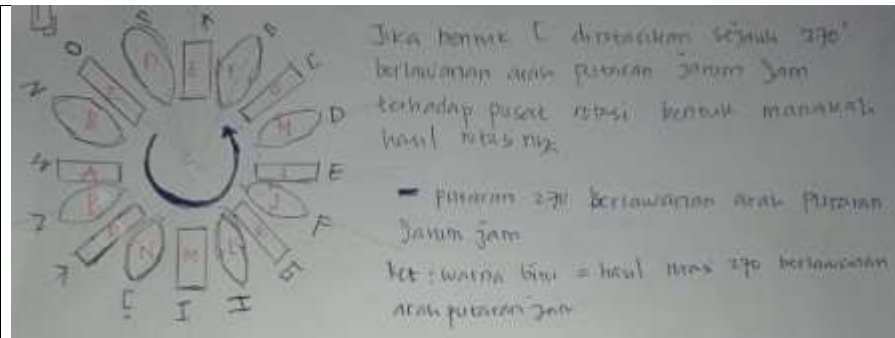
Error Description:
Student A gives the correct answer result. Conceptually, students explain the answer correctly by showing the counterclockwise direction with the formula R = -240º. However, procedurally there is still a lack of detail. Referring to the expected achievement of the second indicator, it is known that student A has not achieved the indicator. Students can understand the ideas of the questions and develop them through answers. The drawback is that it does not explain the answer in detail or interpret in other visual forms to show "counterclockwise". The analogy is when asked for a home address and answered the address according to the one on the identity card (KTP). The answer is not wrong, but it has not clearly shown the location of the house's position. Question number 2 is given in the form of pictures, it will be better and clearer if explained the answer and a picture. This incident can be overcome by getting used to answering detailed questions every day.

Results of the interview and its analysis:
Student A stated that the answer was correct based on the interview results. Students confidently explain the origin of the answer with the correct concept. Uniquely, when asked the same question, student A answered the teacher's question by pointing to the picture on the question. Students understand the intent and direction of the question, but on the answer sheet give a short answer because he thinks the answer is enough. When asked, "why not add a picture to the answer to question number 2", student A replied "because from the question there is no command to draw, and my answer has already answered the question". The results of students' answers are incorrect and still need an explanation confirming the answer.

Student B (Medium category)

If the C shape is rotated by 270o counterclockwise
rotation to the center of rotation, which shape is the result of rotation
⇨ Round 270 counterclockwise rotation
Description: blue color = rotation result 270
in the opposite direction of rotation of the clock

Error Description:
Student B gave the correct answer. On the answer sheet, student B does not write what the answer is but it is shown in the picture that the result of rotation of the C shape as far as 270o counterclockwise is the shape G. Conceptually, student B is correct in describing the rule of rotation counterclockwise as far as 270o. Procedurally, student B has correctly solved the problem in a simple form. Furthermore, technically also student B did not make a mistake. However, procedurally still incomplete, this error occurs because student B does not follow the steps of the procedure which usually includes ending the answer or explanation with a conclusion or summary of the information that has been submitted. They refer to the second indicator of mathematical communication skills, namely interpreting mathematical ideas in writing and other visual forms. Based on the results of students' answers, they still do not meet the point of "being able to interpret mathematical ideas in writing".

Results of the interview and its analysis:
Based on the interview results, student B stated that question number 2 was not too difficult and was sure that what was done was correct. Student B said he had done almost the same problem during practice at home. So question number 2 adds a line to show the size of the rotation. However, when asked "so what is the result of the rotation of the C shape?" the student immediately showed the picture he made. Then asked again "where? Why is it not written with the answer in question?" the student hesitated and expressed his shortcomings "oh yes yes ma'am, I forgot to write it". Student B did not make a mistake or give a final conclusion on the answer sheet. This deficiency of student B can undoubtedly be overcome if students are accustomed to solving story problems in a known writing order, which is asked, the answer, and the conclusion. Students can also be made into discussion groups to complement each other's shortcomings in the question answer results.

Student C (High category)
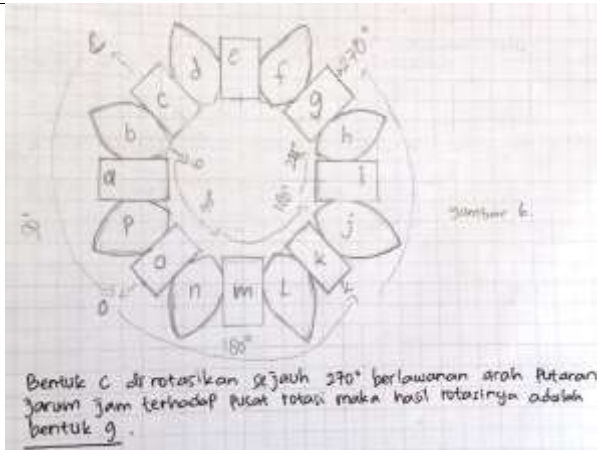
The c shape is rotated as far as 270º counterclockwise rotation to the center of rotation, so the result of rotation is the g shape

Error Description:
Student C has correctly written the answer along with the complete description. Students with high communication skills seem accustomed to communicating their ideas through writing and pictures. The image created is also equipped with clear and easy-to-understand captions.

Results of the interview and its analysis:
Based on the interview results, student C stated that question number 2 is elementary and can quickly solve it. According to student C, questions like this increase their accuracy because they are done quickly so there is plenty of time to check back and tidy up the written answers. Uniquely, student C stated that he was worried if he finished quickly because other students asked a lot. This means that C students lack empathy and sharing, so it would be better to solve questions in groups. So that students who have high abilities can share with group members who have medium or low abilities.

Figure 4. Description of Student Answers Question Number 2 is reviewed from the Student Communication Skills Category

Furthermore, question number 3 represents the third indicator: the ability to use terms, mathematical notations and structures to present ideas and describe relationships and models of situations. The problem is "If the sketch of the JFC costume wing defile Betawi is brought to the coordinate approach by suppose one of the triangular-shaped parts of the wing has coordinates points A(-5.11), B(-1.1), C(-2.12) reflected against the X axis. Determine the coordinates of the shadow of the ABC point!".

The results of students' work solving question number 3 are presented based on the categories of student communication skills (high, medium, and low) randomly selected in the following figure.

Student A (Low category)

Error Description:
Student A writes the correct answer according to what is asked in question number 3 and uses a mirroring formula on the X axis. The hope in question number 3 is to see the student's ability to use terms, mathematical notations and structures to present ideas and describe relationships and models of situations. The facts written by student A have met the abilities in the third indicator, just not yet detailed adding other necessary explanations. This shows that students commit procedural mistakes that cannot solve problems in various simple forms and manipulate them. The third indicator has an expected point that has not been met from student A's answer, namely "also able to describe relationships and situation models". However, student A gives an answer that matches the question given.

Results of the interview and its analysis:
Based on the interview results, student A stated that it was difficult when doing question number 3. The difficulty is caused by the mirroring formula against the X and Y axes, which are almost the same, so they are sometimes reversed. Some students express minor difficulty if the question is negative because sometimes they forget to operate it. However, judging from the results of student answers, students have almost fulfilled the third indicator of mathematical communication skills, namely the ability to use terms, mathematical notations and structures to present ideas and describe relationships and situation models. It lacks the ability point in "describing relationships and situation models". According to students, the questions given have no drawing commands, so students only write down formulas that are remembered when facing problems. This shows that students lack the initiative to double-check the correctness of the answers. So that learning that trains students to recheck the answer sheet can familiarize students to provide several answers or checking steps when solving questions.
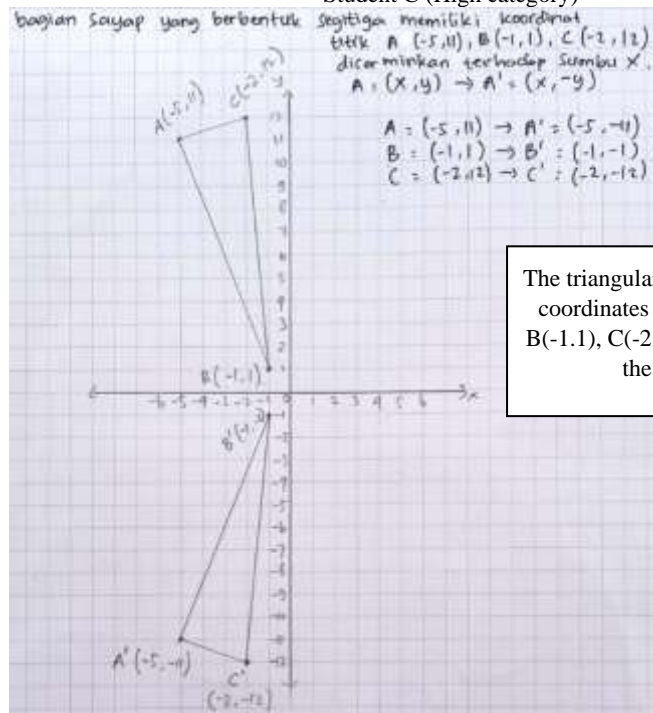
Student B (Medium category)

Error Description:
Student B gives the correct answer and is the same as the answer written by student A. So it is known that student B made a procedural error because it was incomplete in describing relationships and situation models.

Results of the interview and its analysis:

Based on the results of the same answer between student A and student B, the description that can be explained is the same. Student B's answer already showed that the student almost met the third indicator, only lacking in the point of "describing relationships and situation models". However, based on the results of the interview, it was found that student B felt that the answer was not entirely correct. Student B is not yet convinced but is reluctant to double-check with pictures or try to do it differently. Student B stated that if this problem were solved by group discussion then the final answer written would be more convincing. This statement is an input that group discussion is one of the necessary learning stages.

Student C (High category)

bagian sayap yang berbentuk segitiga memiliki koordinat titik A (-5,11), B(-1,1), C(-2,12) dicerminkan terhadap Sumbu X.

$A : (x, y) \rightarrow A' : (x, -y)$

$A = (-5, 11) \rightarrow A' = (-5, -11)$
$B = (-1, 1) \rightarrow B' = (-1, -1)$
$C = (-2, 12) \rightarrow C' = (-2, -12)$

The triangular part of the wing has coordinates of points A(-5.11), B(-1.1), C(-2.12) reflected against the X-axis. 68

Error Description:
Student C writes down the answers correctly and clearly. Question number 3 is based on the third indicator, meaning that student C has fulfilled the third indicator: the ability to use terms, mathematical notations and structures to present ideas and describe relationships and situation models. The answers written are in the form of the correct formula to solve the given problem, and are used correctly so that the results written are also correct. Next, student C provides a picture as a further explanation as well as a cross check to make sure the answer written is correct. The drawings provided accurate explanations and showed the correct questions and answers.

Results of the interview and its analysis:
Based on the interview results, student C stated he was enthusiastic about doing question number 3. This is because in addition to the easy problem, student C also stated that he likes problems that can be solved with pictures. Uniquely, student C stated that he forgot the mirroring formula for the X axis. So the first step taken to solve the problem is to draw it.

Student C then annotates with a formula based on the drawing he made. This can also be seen from the work of students who draw first and then rewrite the results of the answer along with the formula. This fact shows that student C, who have high communication skills, does mathematical communication by presenting mathematical ideas through pictures by looking at relationships and situation models.

Figure 5. Description of Student Answers Question Number 3 is reviewed from the Student Communication Skills Category

Overall, the analysis of student answer sheets and the interview results found that one way to improve students' mathematical communication skills is to minimize the mistakes that students often make. Most mistakes are made by students in the low and medium categories, so the solutions offered are especially to minimize the mistakes made by students in the low and medium categories. Students with low communication skills make some mistakes they are unaware of and tend to make conceptual and procedural mistakes. Students believe the answer is correct and tend not to want to check again. Even though when interviewed and asked to double-check the answer, students realized where they were wrong.

This needs a solution to minimize the mistakes students make when solving problems so that it has an impact on improving students' mathematical communication skills. It is known that some mistakes made by students with low communication skills on average start from conceptual errors that impact procedural errors and continue to technical errors. This certainly impacts the overall wrong answer in solving mathematical communication problems. From the analysis above, one of the steps that can be used is to do repeated exercises, group discussions, provide feedback, get used to double-checking answer sheets, and use concrete material, one of which is using culture during the learning process.

Furthermore, students with moderate communication skills categories were found to be prone to technical mistakes. They mostly expressed doubts about the answers written, so they gave incomplete answers. The results of the answer analysis found that the answers of students with communication skills categories gave correct answers, but wrote the answers irregularly and in less detail. So students like this need an active learning environment doing repetitive and complementary exercises. One solution in learning is to hold group discussions, complete contextual exercises, then provide consistent space to encourage students to check back with the results of the answers.

Observation Results of Ethnomathematically Based Learning Steps assisted by Geogebra Application

The role of ethnomathematics in learning is that students can recognize and use connections between mathematical ideas in solving project problems, relate mathematical and mathematical ideas to disciplines outside mathematics, and mathematics with the natural world in everyday life (Mania & Alam, 2021; Nur et al., 2020). One of the cultures used in learning is the motif on batik. The material chosen is transformation geometry material for 8 meetings (one initial test, one final test, and 6 learning times). The syntax of Ethnomathematics-based learning assisted by the Geogebra application applied in transformation geometry learning is presented in the following figure.

Table 6. Ethnomathematical Based Learning Stage assisted by Geogebra Application

Stages	Activity
Exploration stage	Student activities explore mathematical ideas in culture. Students are given material about culture

Mapping stage	With the assistance of the teacher, students make a map of the relationship between mathematical concepts (transformation geometry) and ethnomathematics
Explanatory stage	Students learn mathematical concepts (transformation geometry) with the help of the Geogebra application and communicate what is learned, share, appreciate what is learned in various forms
Reflection stage	Summarizing what is learned both knowledge of mathematical concepts (geometry of transformations) and cultural values developed in the learning process

Based on the results of observations during learning, it is known that all students are very enthusiastic about each stage of learning given, especially during the mapping stage.

Figure 6. Students Discuss Working on Worksheets

At this mapping stage, the individual learning process becomes a discussion forum. Students are engrossed in asking each other to discuss and draw the relationship between batik motifs and transformation geometry material presented on worksheets. From the observations, students mentioned a lot about extracurricular batik activities that are usually done every Friday. Discussions become more attractive for students and the material delivered is connected to their daily lives so that it becomes more meaningful not just memorizing. Contextual learning and discussion invite students to understand the material not just memorizing (Ozdem-Yilmaz & Bilican, 2020; Sung et al., 2022). Furthermore, the explanation stage is also a stage that significantly impacts students' communication skills.

Figure 7. Students understand the Material with the Geogebra Application

Accurate visuals inspire students to find connections between concepts and objects displayed. Other visual representations help students understand mathematical abstractions better, which in turn can improve their ability to explain those concepts to others. The help of the Geogebra application makes learning mathematics more interactive. Students can interact with mathematical objects in this case batik patterns, then change parameters, and see how those changes affect the results. This helps them feel and understand concepts more deeply, which can translate into better communication skills. Furthermore, the GeoGebra application allows students to collaborate in understanding mathematical concepts (Celen, 2020).

Students share their work with classmates during learning, stimulating them to discuss and exchange ideas. The ability to communicate and convey their understanding to others can

develop in this context. Due to limited computer media in the school where the research is conducted, the use of the Geogebra application is designed with 3 steps, namely explanation by the teacher in front of the class, direct practice by several students in front of the class, and direct practice by students with their respective students' mobile phones that have installed the Geogebra application before learning. At the first meeting, spending time that did not fit the lesson plan because students were adjusting to using the Geogebra application in understanding the material and solving geometry problems. However, at the next meeting the students were enthusiastic and the learning went according to the plan. Students like this kind of learning, the results of direct interviews with students are known that students are bored with ordinary learning. Students state that using apps makes learning less sleepy and very interesting. In addition, learning becomes more interactive and students can practice directly.

Nevertheless, some students mention that expect other variations in learning such as games or group discussions. Students also mentioned that the questions done were less numerous and less varied so that when faced with different questions they became confused and complicated. According to students, sometimes when they have completed a question and are still unsure about the results, students feel reluctant to ask questions or check again. This is undoubtedly an input that needs to be considered to develop an ethnomathematics-based learning model assisted by geogebra applications.

In conclusion, based on the results of interviews and analysis of student answer sheets then strengthened from observations during learning, it is known that Ethnomathematics-based learning with the help of the Geogebra application has been effective for students' mathematical communication skills. However, minimizing the mistakes made by students will further improve students' mathematical communication skills more optimally. As for some of the developed stages are presented in the following table.

Table 7. Development of Ethnomathematical Based Learning Syntax assisted by Geogebra Application

Stages	Activity
Exploration stage	Student activities explore mathematical ideas in culture. Students are given material about culture
Mapping stage	With the assistance of the teacher, students make a map of the relationship between mathematical concepts (transformation geometry) and ethnomathematics
Explanatory stage	Students learn mathematical concepts (transformation geometry) with the help of the Geogebra application and communicate what is learned, share, appreciate what is learned in various forms
Tahap exercise	Using group discussions, students are given practice questions of various types to strengthen understanding of concepts and practice skills
Stages of investigation by way of exchange of group members	Students are directed to re-investigate the results of answers discussed by exchanging group members.
Reflection stage	Summarizing what is learned both knowledge of mathematical concepts (geometry of transformations) and cultural values developed in the learning process

The table above shows two stages added based on the results of data triangulation, as for the two stages, namely the exercise stage and the investigation stage. The exercise stage is to give practice questions of various types done in groups. The results of the previous analysis were known to students in the medium and low mathematical communication skills category,

the majority understood the concept of the problem at hand, but most made procedural errors and technical problems based on lack of practice. In the learning process, it is not uncommon to give practice questions to their students, but it is not a mandatory learning stage so that time runs out with material exposure and a little practice. Through practice, students can better understand how they think and structure their mathematical thinking (Hwang et al., 2021). According to students, practice can help students develop presentation skills, especially if asked to present a solution or problem-solving in front of the class. This is supported by Alam & Mohanty (2023), that teachers can provide constructive feedback regarding students' mathematical communication skills through exercises. Feedback helps students understand areas for improvement and improve the quality of their communication. So, doing exercises needs to be a priority stage by making it one of the stages that must appear in every learning process with a measurable period.

The second stage developed is the investigation stage, which asks students to exchange their group members with other group members, then investigate the results of the exercise discussion. This is done as a step of cross-checking answers but still in discussion. Investigations conducted after exchanging group members become more effective as they are faced with new members and new ideas to investigate the results of answers. Activities involve investigating and comparing students' answers to ascertain correctness or error in solving mathematical problems (Mellone et al., 2020). Cross-checking helps students become more aware of any mistakes or mistakes they may make during the completion process.

Further, this awareness can help students correct their understanding and avoid similar mistakes in the future. According to Reid O'Connor & Norton (2022), by examining the answers thoroughly, students are more likely to focus on the completion process rather than just looking for the final answer. It helps students in the development of logical thinking skills and systematic steps. Re-checking activities can involve cooperation between students in learning. Students can also learn from classmates' mathematical approaches and thinking, enriching their learning experience. With the above verification, the system developed from this study not only improves students' mathematical communication skills, but also guides students to have more meaningful learning behaviors in mathematics. Through deeper understanding and better problem solving, students can gain confidence in facing similar math tasks in the future.

CONCLUSION

It was concluded that students of medium and low mathematical communication skills made more procedural errors. By minimizing errors, it directly improves students' mathematical communication skills. The solution found to minimize student errors is to make the exercise stage and the investigation stage as stages that must appear every Ethnomathematics-based learning. So that the stages of Ethnomathematics-based learning developed consist of exploration, mapping, explanation, exercise, investigation, and practical reflection on students' mathematical communication skills. For further research, it is necessary to conduct experimental research to test the effectiveness of the Ethnomathematics-based learning model developed on students' mathematical communication skills.

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Dear Author/s: Hamidah[1], Jaka Wijaya Kusuma[2]

Affiliation: [1,2] Universitas Bina Bangsa, Serang, Indonesia

We are pleased to inform you that the abovementioned manuscript has been reviewed and accepted for publication in the upcoming Regular issue (2024) of the **Advances in Nonlinear Variational Inequalities, ISSN** 1092-910X. This letter of acceptance is to be considered as the official acceptance of your manuscript with no further amendments required. Author/s are requested to follow ethical and privacy guidelines mentioned in the journal homepage

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